COMPUTATIONAL VAMEDECUM OF THE COUPLED MECHANICAL/THERMAL BEHAVIOR OF COMPOSITE MATERIALS DURING ULTRASONIC WELDING

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Abstract. Aiming faster and more reliable end products, the composite material industry is nowadays an active research topic. Innovative composite forming processes are actively designed and tested. For example, ultrasonic welding of composite thermoplastic materials is being investigated, since it shows many advantages over classical methods. In fact, energy directors allow a preferential heating of the manufactured part through the propagation of mechanical waves in a composite laminate, without including any foreign material in the welded region. However, ultrasonic welding of composite materials is not mastered yet because of the coupled and complex behavior of such materials. Thus, simulation of ultrasonic heating becomes compulsory for understanding the complex multi-physics coupled problem.

In this work, we propose to model the ultrasonic welding process using a dynamic viscoelastic model in the frequency domain. Later on, this model is coupled to the transient

heat equation, giving the temperature field as well as the heat flux in the simulated part. However, the result depends on the chosen experimental and material parameters such as the thickness of the part, its viscosity, its modulus of elasticity, the imposed frequency and displacement... Which makes the optimization of the process a tricky issue requiring a new set of solutions of the problem for each choice of the process parameters.

Using the proper generalized decomposition (PGD), along with a coupled viscoelastic/thermal model, where all the parameters mentioned above are included as extra coordinates of the problem, appears to be a suitable solution for the optimization problem. Moreover, the PGD multidimensional solution considering all the process parameters as extra coordinates is obtained within a realistic timeframe. In fact, by using the PGD, we alleviate the curse of dimensionality since the PGD performs a separation of variables which reduces the problem dimensionality [1]. The result is therefore a computational vademecum that can be used to explore in real time the solution of the problem for any choice of the process parameters, speeding up its optimization [2].

1 INTRODUCTION

Ultrasonic welding is a promising technique for joining composite thermoplastic parts, using mechanical vibration transversally applied to the material. In fact, ultrasonic welding is nowadays one of the most popular methods for joining unreinforced thermoplastic materials in a wide range of industries including automotive, biomedical, electronics [3, 4]... In fact ultrasonic welding features many benefits like high speed welding performed within a fraction of a second to few seconds, and without using any foreign materials in the welded region. However, ultrasonic welding of fiber reinforced thermoplastic composite materials is not popular or industrially applied yet [3], despite the promising results published in previous research works [5, 6, 7]. This can be explained by the lack of understanding of the physics occurring in the process like mechanical strain propagation and heating [3].

Therefore, the simulation of ultrasonic welding of composite thermoplastics becomes compulsory to help the industry understands the coupled multi-physics occurring in the welded part, which may validate the currently used techniques. However, many process and material parameters affect the quality of the weld. Several authors investigated the process parameters independently on a given material [8, 9]. In this work, we choose to include the relevant process and material parameters into the governing equation of the problem as extra coordinates of the problem. For instance, the thickness of the part, its viscosity, the modulus of elasticity, the imposed frequency and displacement amplitude are included as extra coordinates of the problem. This leads to a "complete" solution covering all the possible combinations of parameters, a "computational vademecum". Afterwards, the user may optimize the process using any classical optimization algorithm

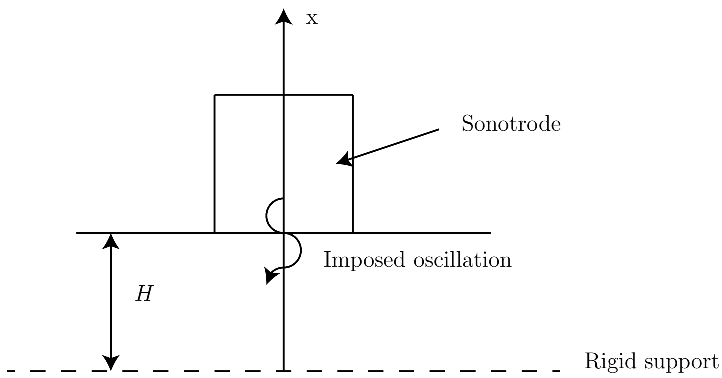

Figure 1: Simulated model

like Newton of Levenberg-Marquardt algorithms since the gradient with respect to the parameters are readily available [10, 11].

However, adding extra coordinates to the problem increases its dimensionality and therefore leads to unrealistic calculation time while using classical simulation techniques. Using the PGD, we can alleviate the curse of dimensionality by decomposition the high dimensionality problem into a sequence of lower dimensionality problems. For instance, one may solve a 3D problem as a sequence of 1D problems or 2D, 1D problems [1]. In this work, we compute a coupled mechanical/thermal vademecum adding five extra coordinates to the space-time dimensions.

In section 2, we review the used Kelvin-Voigt mechanical model to simulate the process, with the PGD construction of the mechanical solution. Later on, section 3 shows the thermal simulation model and its construction using the PGD. In section 4, we compare the obtained vademecum to the solutions obtained using classical algorithms and discuss the result. Finally we draw some conclusions in section 5.

2 MODELING THE STRAIN FIELD IN ULTRASONIC WELDING

In this section we model the ultrasonic welding of thermoplastic materials. The chosen model is illustrated in figure 1. We choose a part of thickness H, where a mechanical vibration is imposed by the sonotrode on the top of the domain. As a mechanical model, we choose the Kelvin-Voigt viscoelastic model which can be represented by a pure elastic spring connected in parallel to a pure viscous damper, thus the stress tensor in the material σ is written as:

$$\sigma = \sigma_{elastic} + \sigma_{viscous} = E \cdot \epsilon + \eta \dot{\epsilon} \tag{1}$$

where E is the modulus of elasticity of the used material, η is its viscosity, ϵ the strain tensor and $\dot{\epsilon}$ the rate of strain tensor. Working in the frequency domain, one may write the strain as:

$$\epsilon = \epsilon_0 \cdot e^{iwt} \tag{2}$$

w being the angular frequency in rad/s, i the imaginary number and t the time. Replacing ϵ in Eq. (1) leads to:

$$\sigma = \sigma_0 \cdot e^{iwt-i\theta} \tag{3}$$

with $\sigma_0 \cdot e^{-i\theta}$ given by:

$$\sigma_0 \cdot e^{-i\theta} = (E + iw\eta)\,\epsilon_0 \tag{4}$$

Considering the dynamics equation:

$$\rho \ddot{u} = \nabla \cdot \sigma \tag{5}$$

ρ being the density and ü the displacements, and replacing (3) into (5), rearranging leads to the problem governing equation. For the sake of simplicity and without any loss of generality, we choose to simulate the process in a unidirectional physical domain $x \in [0, H]$ representing the thickness of the welded part. Therefore the governing equation is written by:

$$\begin{cases} -\rho U w^2 = G^* \cdot \frac{\partial^2 U}{\partial x^2} \\ U = U_r + i \cdot U_i \\ G^* = G' + i \cdot G'' = E + i \cdot w \cdot \eta \end{cases} \tag{6}$$

U being the amplitude of the displacement. The boundary conditions of the problem are:

$$\begin{cases} U(x=0) = 0 \\ U(x=H) = U_0 \end{cases} \tag{7}$$

Eq. (6) is defined in the complex domain, where U and G both have real and imaginary components. The displacement U is solved considering the thickness of the part H, its viscosity η, the modulus of elasticity E, the imposed frequency w and displacement amplitude U_0 as extra coordinates of the problem. Therefore, we will introduce these parameters into the differential equation of the problem written in Eq.(6).

One may note that η, E and w appears naturally in the differential equation, while U_0 appears in the boundary conditions. These parameters can be easily introduced as coordinates of the problem as thoroughly discussed in previous publications [1, 12, 2]. However, introducing the geometrical parameter H as extra coordinate of the problem is slightly more elaborated [13, 14]. First we define the mapping between the real domain of length H and a domain $s \in [0; 1]$ written by:

$$x = H \cdot s \tag{8}$$

Then replacing Eq. (8) into Eq. (6), one may find the integral form of the problem to solve defined in the s domain by:

$$\int_{\Omega} U^* \cdot \left(\rho U w^2 - G^* \cdot \frac{\partial^2 U}{\partial s^2} \cdot \frac{1}{H^2} \right) H \cdot ds \cdot dE \cdot dH \cdot d\eta \cdot dw \cdot dU_0 = 0 \tag{9}$$

The boundary conditions are now written by:

$$\begin{cases} U(s=0) = 0 \\ U(s=1) = U_0 \end{cases} \tag{10}$$

The problem defined in Eq. (9) can be solved using the PGD with a fixed point, rank one update greedy algorithm as detailed in different publications [1, 10]. The solution is given in a separated form as:

$$U = \sum_{j=1}^{j=N} S_j(s) \cdot \mathcal{H}_j(H) \cdot N_j(\eta) \cdot \mathcal{E}_j(E) \cdot W_j(w) \cdot \mathcal{U}_j(U_0) \tag{11}$$

For this problem, the PGD algorithm converges within less than a minute, at a number of product of functions $N = 25$. Once $U(x, H, \eta, E, w, U_0)$ is computed, one may compute the strain ϵ in a separated form by:

$$\epsilon = \sum_{j=1}^{j=N} \frac{\partial S_j(s)}{\partial s} \cdot \frac{\partial s}{\partial x} \cdot \mathcal{H}_j(H) \cdot N_j(\eta) \cdot \mathcal{E}_j(E) \cdot W_j(w) \cdot \mathcal{U}_j(U_0) \tag{12}$$

where $\frac{\partial s}{\partial x} = \frac{1}{H}$ as per Eq. (8). As one may notice, all the derivatives with respect to all the chosen parameters are now readily available using Eq. (11).

3 MODELING THE THERMAL PROBLEM USING THE PGD

Once $U(x, H, \eta, E, w, U_0)$ and $\epsilon(x, H, \eta, E, w, U_0)$ are available, one may compute the dissipated power through the studied part, defined as:

$$Q = w \eta \epsilon^2 \tag{13}$$

Which lead us to a heat source Q written in a separated form by:

$$Q = w \cdot \eta \cdot \left[\sum_{j=1}^{j=N} \frac{\partial S_j(s)}{\partial s} \cdot \frac{1}{H} \cdot \mathcal{H}_j(H) \cdot N_j(\eta) \cdot \mathcal{E}_j(E) \cdot W_j(w) \cdot \mathcal{U}_j(U_0) \right]^2 \tag{14}$$

$E \in [2, 5]\, GPa$
$\eta \in [4000, 8000]\, Pa.s$
$H \in [10^{-4}, 5 \times 10^{-4}]\, m$
$w \in 2\pi \times [20000, 50000]\, rad/s$
$U_0 \in [1, 10]\, mm$

Table 1: Chosen intervals of variation of the parameters

Later on, we solve the transient heat equation written by:

$$\rho \cdot C \cdot \frac{\partial T}{\partial t} - K\frac{\partial^2 T}{\partial x^2} = Q \tag{15}$$

where C is the heat capacity of the studied composite material, K its conductivity and T the temperature. Writing the integral form of Eq. (15) in the parametric domain after applying the geometrical transformation written in Eq. (8) gives:

$$\int_\Omega T^* \left(\rho \cdot C\frac{\partial T}{\partial t} - K\frac{\partial^2 T}{\partial s^2} \cdot \frac{1}{H^2} - Q\right) H \cdot ds \cdot dE \cdot dH \cdot d\eta \cdot dw \cdot dU_0 = 0 \tag{16}$$

One may wish to add the conductivity K and the heat capacity C as extra coordinates of the problem. For the sake of simplicity, these 2 parameters are kept as constants in the numerical example shown in section 4. The boundary conditions for the heat problem are given by:

$$\begin{cases} -k\frac{\partial T}{\partial x} = 0 \text{ at } x = 0 \\ \\ -k\frac{\partial T}{\partial x} = h(T - T_{out}) \text{ at } x = H \end{cases} \tag{17}$$

We are therefore imposing convective boundary conditions on the top surface of the domain $x = H$ and an adiabatic boundary condition at the bottom surface at $x = 0$. T_{out} is the outside temperature set to be 293K for the illustrative example shown in section 4 and $h = 25\ W/m^2.K$.

4 NUMERICAL EXAMPLE, VERIFICATION AND DISCUSSION

In the following section, we illustrate a numerical example of ultrasonic welding of a composite material having a density $\rho = 1300\ kg/m^3$, the domains of variation of the parameters are given in table 1.

In figure 2 we illustrate the variation of the displacement amplitude as a function of the position x for the following combination of parameters: E=2 GPa, η=4000 Pa.s, H=0.1 mm, w=2$\pi\times$20000 rad/s and $U_0 = 1$ mm. We notice that the displacement amplitude is linear function of x since the height of the domain is small with respect to the oscillation period, which is about 15 mm considering the speed of the mechanical wave about 300 m/s and the chosen frequency is 20 000 Hz. Figure 3 illustrates a finite difference solution for the same combination of the parameters. The relative error between the two solutions doesn't exceed 0.4%.

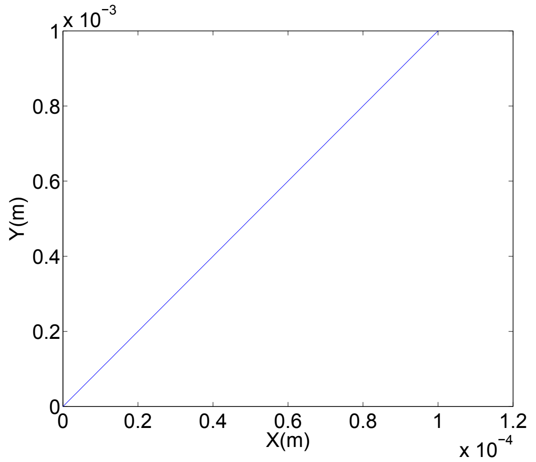

Figure 2: Displacement amplitude as a function of the position x obtained from the computed PGD vademecum

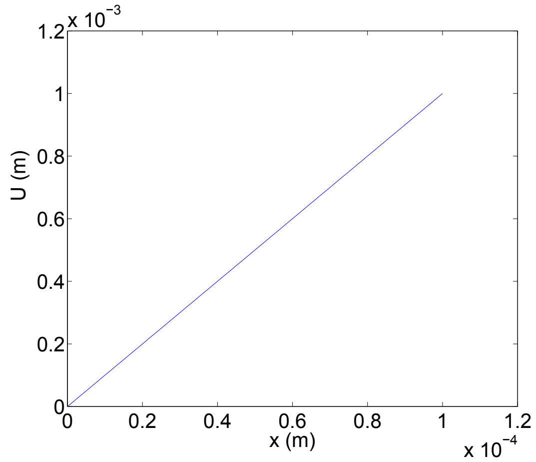

Figure 3: Displacement amplitude as a function of the position x obtained by the finite difference method

Since the displacement amplitude is linear as a function of the thickness of the part, the strain and therefore the heat source Q are nearly constant for the chosen value of H.

In figure 4, we are illustrating the displacement amplitude $||\vec{U}||$ in the middle of the domain at $x = H/2$, as a function of three chosen parameter, E, w and η. In figure 5, we are illustrating the heat source Q as a function of these parameters at the same position. One may notice that the heat source is sensitive to the variation of w and η, but is less sensitive to the changes of the modulus of elasticity of the material E for example. A similar study for H and U_0 shows that the heat source Q changes function of the variations of H, U_0, w and η but less sensitive to the variation of E.

Once the heat source is computed, we solve the transient heat equation for a convective boundary condition on the top of the domain considering $h = 25\ W/m^2.K$, the heat capacity $C = 1450\ J/kg.K$ and a conductivity $K = 0.24\ W/m.K$. Figure 6 illustrates the temperature field in the part as a function of the position x and the time t for E=2 GPa, η=4000 Pa.s, H=0.1 mm, w=$2\pi\times$20000 rad/s and $U_0 = 1$ mm.

Figure 7 illustrates the temperature field for the chosen parameters at $t = 10$ s, while figure 8 illustrates the temperature fields for the same combination of parameters obtained by the finite differences method at $t = 10$ s. The relative error between the two fields is shown in figure 9. We note that the relative error does not exceed 1.5%. This error

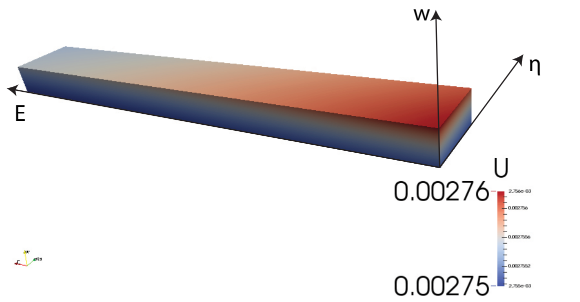

Figure 4: Displacement amplitude as a function of the parameters E, w and η at $x = H/2$

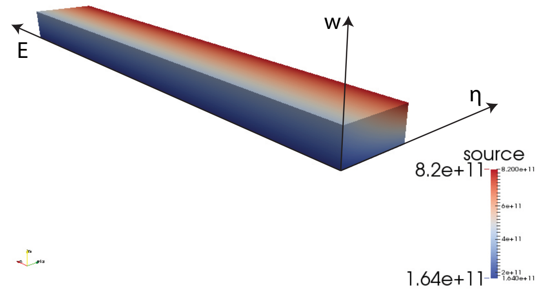

Figure 5: Heat source Q as a function of the parameters E, w and η at $x = H/2$

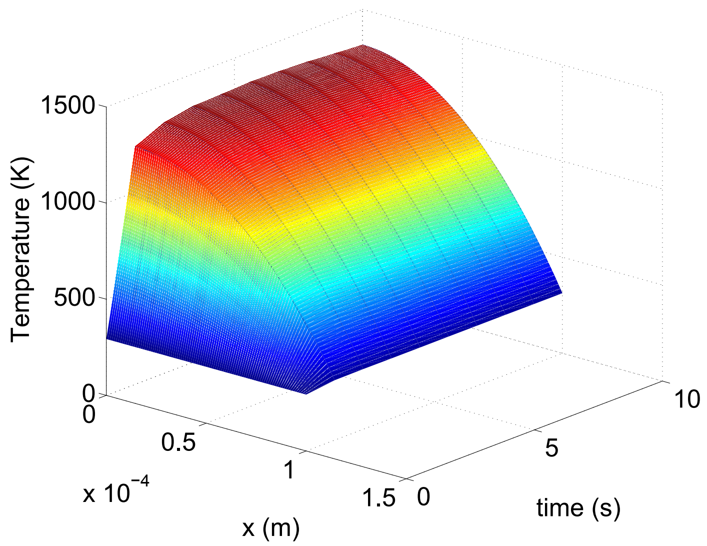

Figure 6: Temperature field in the studied part for the chosen combination of parameters

is larger than the one obtained for the displacement amplitude since the strains are the derivatives of the displacements and therefore the heat source error is generated by the derivatives of U, which is normally larger than the error on U. This error can be reduced by pushing further the PGD convergence for U by increase the value of N beyond 25.

5 CONCLUSIONS

In this work, we illustrate an approach to simulate the temperature fields during ultrasonic welding of composite materials by considering the process and material parameters as extra-coordinates of the problem within realistic time frame. First, we illustrate the coupled viscoelastic mechanical and thermal problems. In our model, we use the Kelvin-Voigt viscoelasticity coupled to the heat equation. The result is a computational vademecum that can be used to optimize the process on the fly, in real time [11].The

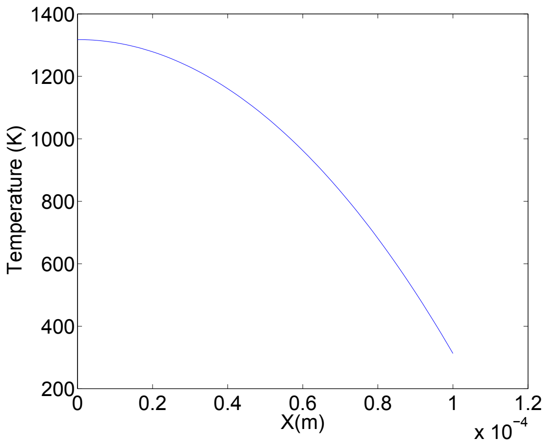

Figure 7: Temperature fields at t=10 s obtained from the PGD solution

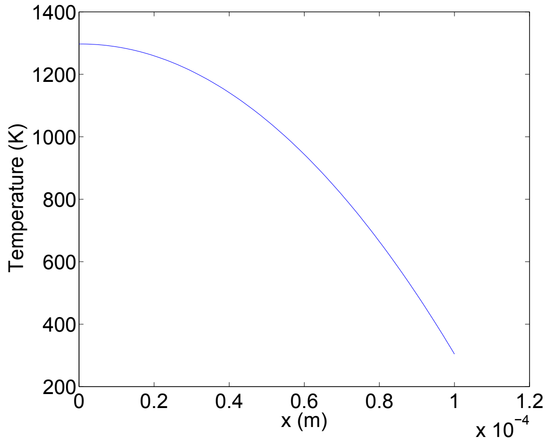

Figure 8: Temperature fields at t=10 s obtained from the finite differences solution at the same combination of parameters

obtained results are verified by comparison to the ones computed by classical algorithms and the resulting relative error is acceptable. The obtained vademecum can also be used to compare to experimental results and identify materials parameters or optimize the process.

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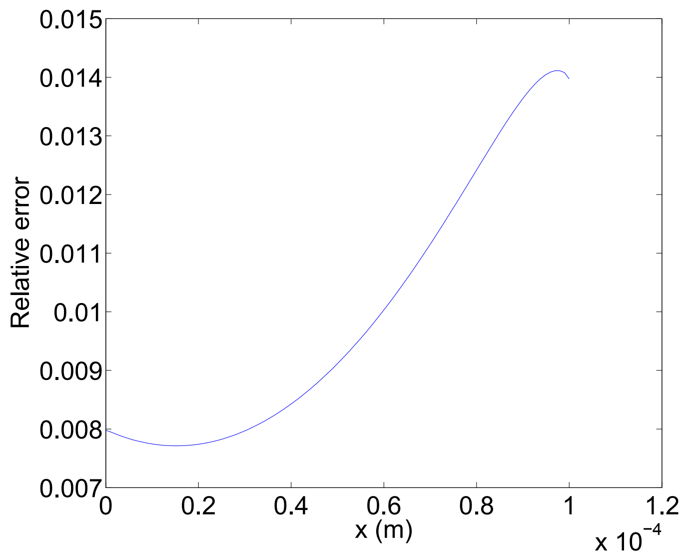

Figure 9: Relative error between the PGD and finite difference solutions illustrated in figure 7 and 8

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